

Phases of Flavor Neutrino Masses and CP Violation

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For flavor neutrino masses M_{ij}^{PDG} ($i, j = e, \mu, \tau$) compatible with the phase convention defined by Particle Data Group (PDG), if neutrino mixings are controlled by small corrections to those with $\sin \theta_{13}=0$ denoted by $\sin \theta_{13} \delta M_{e\tau}^{PDG}$ and $\sin \theta_{13} \delta M_{\tau\tau}^{PDG}$, CP-violating Dirac phase δ_{CP} is calculated to be $\delta_{CP} \approx \arg[(M_{\mu\tau}^{PDG*} / \tan \theta_{23} + M_{\mu\mu}^{PDG*}) \delta M_{e\tau}^{PDG} + M_{ee}^{PDG} \delta M_{e\tau}^{PDG*} - \tan \theta_{23} M_{e\mu}^{PDG} \delta M_{\tau\tau}^{PDG*}] \pmod{\pi}$, where θ_{ij} ($i, j=1,2,3$) denotes an i - j neutrino mixing angle. If possible neutrino mass hierarchies are taken into account, the main source of δ_{CP} turns out to be $\delta M_{e\tau}^{PDG}$ except for the inverted mass hierarchy with $\tilde{m}_1 \approx -\tilde{m}_2$, where $\tilde{m}_i = m_i e^{-i\varphi_i}$ ($i=1,2$) stands for a neutrino mass m_i accompanied by a Majorana phase φ_i , where $\varphi_{1,2,3}$ give two CP-violating Majorana phases. We can further derive that $\delta_{CP} \approx \arg(M_{e\mu}^{PDG}) - \arg(M_{\mu\mu}^{PDG})$ with $\arg(M_{e\mu}^{PDG}) \approx \arg(M_{e\tau}^{PDG})$ for the normal mass hierarchy and $\delta_{CP} \approx \arg(M_{ee}^{PDG}) - \arg(M_{e\tau}^{PDG}) + \pi$ for the inverted mass hierarchy with $\tilde{m}_1 \approx \tilde{m}_2$. For specific flavor neutrino masses M_{ij} , whose phases are associated with $M_{e\mu, e\tau, \tau\tau}$, we obtain that $\arg(M_{e\mu}) = \arg(M_{e\mu}^{PDG}) - [\arg(M_{ee}^{PDG}) + \arg(M_{\mu\mu}^{PDG})] / 2$, $\arg(M_{e\tau}) = \arg(M_{e\tau}^{PDG}) - [\arg(M_{ee}^{PDG}) - \arg(M_{\mu\mu}^{PDG})] / 2$ and $\arg(M_{\tau\tau}) = \arg(M_{\tau\tau}^{PDG}) + \arg(M_{\mu\mu}^{PDG}) - 2 \arg(M_{\mu\tau}^{PDG})$. Numerical analysis suggests that Dirac CP-violation becomes maximal as $|\arg(M_{e\mu})|$ approaches to $\pi/2$ for the inverted mass hierarchy with $\tilde{m}_1 \approx \tilde{m}_2$ and for the degenerate mass pattern satisfying the inverted mass ordering and that Majorana CP-violation becomes maximal as $|\arg(M_{\tau\tau})|$ approaches to its maximal value around 0.5 for the normal mass hierarchy.

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Various experimental evidences of neutrino oscillations provided by the atmospheric [1], solar [2, 3], reactor [4, 5] and accelerator [6] neutrino oscillation experiments have indicated that neutrinos have tiny masses and their flavor states are mixed with each other. Nowadays, to study CP violation in neutrinos is one of the important issues to be addressed in order to further understand neutrino physics. The recent observation on the nonvanishing reactor neutrino mixing [5] has opened the possibility that details of Dirac CP-violation can be experimentally clarified in near future. Theoretically, effects of CP-violation are described in terms of three phases, one CP-violating Dirac phase δ_{CP} and two CP-violating Majorana phases $\phi_{2,3}$ [7]. Neutrino mixings are parameterized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary matrix U_{PMNS} [8], which converts the massive neutrinos ν_i ($i = 1, 2, 3$) into the flavor neutrinos ν_f ($f = e, \mu, \tau$). The standard description of U_{PMNS} adopted by the Particle Data Group (PDG) [9] is given by $U_{PMNS}^{PDG} = U_\nu^0 K^0$ with

$$U_\nu^0 = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{CP}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix},$$

$$K^0 = \text{diag}(1, e^{i\phi_2/2}, e^{i\phi_3/2}), \quad (1)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ with θ_{ij} representing a ν_i - ν_j mixing angle ($i, j=1,2,3$). The best fit values of the observed results in the case of the normal mass ordering are summarized as [10]:

$$\Delta m_{21}^2 [10^{-5} \text{ eV}^2] = 7.62 \pm 0.19, \quad \Delta m_{31}^2 [10^{-3} \text{ eV}^2] = 2.53^{+0.08}_{-0.10}, \quad (2)$$

$$\sin^2 \theta_{12} = 0.320^{+0.015}_{-0.017}, \quad \sin^2 \theta_{23} = 0.49^{+0.08}_{-0.05}, \quad \sin^2 \theta_{13} = 0.026^{+0.003}_{-0.004}, \quad (3)$$

$$\frac{\delta_{PC}}{\pi} = 0.83^{+0.54}_{-0.64}, \quad (4)$$

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where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ with m_i representing a mass of ν_i ($i = 1, 2, 3$). The quoted values in the case of the inverted mass ordering ($\Delta m_{31}^2 < 0$) are not so different from Eqs.(2)-(4). There is another similar analysis with Δm_{23}^2 defined as $\Delta m_{23}^2 = m_3^2 - (m_1^2 + m_2^2)/2$ that has reported the slightly smaller values of $\sin^2 \theta_{23} = 0.37 - 0.43$ [11].

In this note, we would like to address the issue of leptonic CP-violation with the emphasis laid on the role of phases of flavor neutrino masses and to find possible correlations between phases of flavor neutrino masses and δ_{CP} and $\phi_{2,3}$ of CP-violation. CP-violating phases arise from complex flavor neutrino masses. However, because of the freedom of choosing charged-lepton phases, phases of neutrino masses are not uniquely defined. Namely, different phase structure gives the same effects of CP-violation. We first discuss how to relate phases of flavor neutrino masses to observed quantities. To do so, we use a neutrino mass matrix M^{PDG} , whose phases are so chosen that the corresponding eigenvectors giving U_{PMNS} show the phase convention defined by PDG, which is nothing but Eq.(1). Next, we give theoretical and numerical estimation of phases of flavor neutrino masses and present possible correlations to CP-violating phases.

We start with discussions based on M^{PDG} defined to be:

$$M^{PDG} = \begin{pmatrix} M_{ee}^{PDG} & M_{e\mu}^{PDG} & M_{e\tau}^{PDG} \\ M_{e\mu}^{PDG} & M_{\mu\mu}^{PDG} & M_{\mu\tau}^{PDG} \\ M_{e\tau}^{PDG} & M_{\mu\tau}^{PDG} & M_{\tau\tau}^{PDG} \end{pmatrix}. \quad (5)$$

Since δ_{CP} is associated with $\sin \theta_{13}$, it is useful to divide M^{PDG} into two pieces consisting of $M_{\theta_{13}=0}^{PDG}$ giving $\sin \theta_{13} = 0$ and ΔM^{PDG} inducing $\sin \theta_{13} \neq 0$ [12]:

$$M^{PDG} = M_{\theta_{13}=0}^{PDG} + \Delta M^{PDG}, \quad (6)$$

with

$$\begin{aligned} M_{\theta_{13}=0}^{PDG} &= \begin{pmatrix} M_{ee}^{PDG} & M_{e\mu}^{PDG} & -t_{23}M_{e\mu}^{PDG} \\ M_{e\mu}^{PDG} & M_{\mu\mu}^{PDG} & M_{\mu\tau}^{PDG} \\ -t_{23}M_{e\mu}^{PDG} & M_{\mu\tau}^{PDG} & M_{\mu\mu}^{PDG} + \frac{1-t_{23}^2}{t_{23}}M_{\mu\tau}^{PDG} \end{pmatrix}, \\ \Delta M^{PDG} &= \begin{pmatrix} 0 & 0 & M_{e\tau}^{PDG} + t_{23}M_{e\mu}^{PDG} \\ 0 & 0 & 0 \\ M_{e\tau}^{PDG} + t_{23}M_{e\mu}^{PDG} & 0 & M_{\tau\tau}^{PDG} - \left(M_{\mu\mu}^{PDG} + \frac{1-t_{23}^2}{t_{23}}M_{\mu\tau}^{PDG}\right) \end{pmatrix}. \end{aligned} \quad (7)$$

It should be noted that Eq.(6) is just an identity. There are specific models giving $M_{\theta_{13}=0}^{PDG}$ [13–16], whose predictions on CP-violation can be covered by our discussions.

Noticing that $M^{PDG} = U_{PMNS}^* M_{mass} U_{PMNS}^\dagger$, where $M_{mass} = \text{diag.}(m_1, m_2, m_3)$, we can express M_{ij}^{PDG} in terms of masses, mixing angles and phases including three Majorana phases $\varphi_{1,2,3}$ that gives $\phi_i = \varphi_i - \varphi_1$. Since $\sin \theta_{13}$ acts as a correction parameter, ΔM^{PDG} is redefined to be $\sin \theta_{13} \delta M^{PDG}$:

$$\begin{aligned} \sin \theta_{13} \delta M_{e\tau}^{PDG} &= M_{e\tau}^{PDG} + t_{23}M_{e\mu}^{PDG}, \\ \sin \theta_{13} \delta M_{\tau\tau}^{PDG} &= M_{\tau\tau}^{PDG} - \left(M_{\mu\mu}^{PDG} + \frac{1-t_{23}^2}{t_{23}}M_{\mu\tau}^{PDG}\right), \end{aligned} \quad (8)$$

from which $\delta M_{e\tau}^{PDG}$ and $\delta M_{\tau\tau}^{PDG}$ are calculated to be:

$$\begin{aligned} \delta M_{e\tau}^{PDG} &= \frac{c_{13}}{c_{23}} [e^{i\delta_{CP}} \tilde{m}_3 - e^{-i\delta_{CP}} (c_{12}^2 \tilde{m}_1 + s_{12}^2 \tilde{m}_2)], \\ \delta M_{\tau\tau}^{PDG} &= \frac{c_{12}s_{12}}{s_{23}c_{23}} e^{-i\delta_{CP}} (\tilde{m}_2 - \tilde{m}_1), \end{aligned} \quad (9)$$

where $\tilde{m}_i = m_i e^{-i\varphi_i}$ ($i = 1, 2, 3$). To estimate CP-violating Dirac phase, let us consider $\mathbf{M} = M^{PDG\dagger} M^{PDG}$. The quantity of $s_{23}\mathbf{M}_{e\mu} + c_{23}\mathbf{M}_{e\tau}$ corresponding to $\Delta M_{e\tau}^{PDG} (= M_{e\tau}^{PDG} + t_{23}M_{e\mu}^{PDG})$ is also known to vanish at $\theta_{13} = 0$ [17]. In fact, it is expressed in terms of observed masses and mixing angles to be:

$$s_{23}\mathbf{M}_{e\mu} + c_{23}\mathbf{M}_{e\tau} = c_{13}s_{13}e^{-i\delta_{CP}} [m_3^2 - (c_{12}^2 m_1^2 + s_{12}^2 m_2^2)]. \quad (10)$$

On the other hand, Eq.(6) yields

$$\begin{aligned} s_{23}\mathbf{M}_{e\mu} + c_{23}\mathbf{M}_{e\tau} &= s_{13}c_{23}^2 \left[\left(\frac{1}{t_{23}} M_{\mu\tau}^{PDG} + M_{\mu\mu}^{PDG} + s_{13}\delta M_{\tau\tau}^{PDG} \right) \delta M_{e\tau}^{PDG*} \right. \\ &\quad \left. + M_{ee}^{PDG*} \delta M_{e\tau}^{PDG} - t_{23}M_{e\mu}^{PDG*} \delta M_{\tau\tau}^{PDG} \right]. \end{aligned} \quad (11)$$

Since $s_{13}\delta M_{\tau\tau}^{PDG}\delta M_{e\tau}^{PDG*}$ in Eq.(11) can be safely neglected, CP-violating Dirac phase δ_{CP} is approximated to be:

$$\delta_{CP} \approx \arg \left[\left(\frac{1}{t_{23}} M_{\mu\tau}^{PDG*} + M_{\mu\mu}^{PDG*} \right) \delta M_{e\tau}^{PDG} + M_{ee}^{PDG} \delta M_{e\tau}^{PDG*} - t_{23} M_{e\mu}^{PDG} \delta M_{\tau\tau}^{PDG*} \right], \quad (12)$$

where an extra π should be added to δ_{CP} if $m_3^2 - (c_{12}^2 m_1^2 + s_{12}^2 m_2^2) < 0$.

To discuss more about δ_{CP} , since contributions of flavor neutrino masses to δ_{CP} depend on their magnitudes, we may include various constraints on M_{ij}^{PDG} supplied by mass hierarchies: $m_{1,2,3}^2$: $m_1^2 < m_2^2 \ll m_3^2$ as normal mass hierarchy, $m_3^2 \ll m_1^2 < m_2^2$ as inverted mass hierarchy and $m_1^2 < m_2^2 \sim m_3^2$ as degenerate mass pattern with $m_1^2 < m_2^2 \approx m_3^2$ (or $m_3^2 \approx m_1^2 < m_2^2$). The magnitudes of masses are controlled by the ideal case of $\theta_{13} = 0$ since corrections to the ideal case are $\mathcal{O}(\sin^2 \theta_{13})$ [18]. For $\theta_{13} = 0$, we have the following estimates of three masses and two mixing angles:

$$\begin{aligned} \tilde{m}_1 &= \frac{c_{12}^2 M_{ee}^{PDG} - s_{12}^2 (M_{\mu\mu}^{PDG} - t_{23} M_{\mu\tau}^{PDG})}{c_{12}^2 - s_{12}^2} = \frac{M_{ee}^{PDG} + M_{\mu\mu}^{PDG} - t_{23} M_{\mu\tau}^{PDG}}{2} - \frac{M_{e\mu}^{PDG}}{c_{23} \sin 2\theta_{12}}, \\ \tilde{m}_2 &= \frac{c_{12}^2 (M_{\mu\mu}^{PDG} - t_{23} M_{\mu\tau}^{PDG}) - s_{12}^2 M_{ee}^{PDG}}{c_{12}^2 - s_{12}^2} = \frac{M_{ee}^{PDG} + M_{\mu\mu}^{PDG} - t_{23} M_{\mu\tau}^{PDG}}{2} + \frac{M_{e\mu}^{PDG}}{c_{23} \sin 2\theta_{12}}, \\ \tilde{m}_3 &= M_{\mu\mu}^{PDG} + \frac{1}{t_{23}} M_{\mu\tau}^{PDG}, \end{aligned} \quad (13)$$

and

$$\tan \theta_{23} = -\frac{M_{e\tau}^{PDG}}{M_{e\mu}^{PDG}}, \quad \tan 2\theta_{12} = \frac{2}{c_{23}} \frac{M_{e\mu}^{PDG}}{M_{\mu\mu}^{PDG} - t_{23} M_{\mu\tau}^{PDG} - M_{ee}^{PDG}}. \quad (14)$$

We are, then, allowed to use the following gross structure of $M_{\theta_{13}=0}^{PDG}$ [19]:

$$M_{\theta_{13}=0}^{PDG} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1/t_{23} \\ 0 & 1/t_{23} & 1/t_{23}^2 \end{pmatrix} M_{\mu\mu}^{PDG}, \quad (15)$$

for the normal mass hierarchy (NMH) [12], and

$$M_{\theta_{13}=0}^{PDG} \approx \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -t_{23} \\ 0 & -t_{23} & t_{23}^2 \end{pmatrix} M_{\mu\mu}^{PDG}, \quad (16)$$

for the inverted mass hierarchy with $\tilde{m}_1 \approx \tilde{m}_2$ (IMH-1) [14], and

$$M_{\theta_{13}=0}^{PDG} \approx \begin{pmatrix} -2 & -2c_{23} \tan 2\theta_{12} & 2s_{23} \tan 2\theta_{12} \\ -2c_{23} \tan 2\theta_{12} & 1 & -t_{23} \\ 2c_{23} \tan 2\theta_{12} & -t_{23} & t_{23}^2 \end{pmatrix} M_{\mu\mu}^{PDG}, \quad (17)$$

for the inverted mass hierarchy with $\tilde{m}_1 \approx -\tilde{m}_2$ (IMH-2) [20], and

$$M_{\theta_{13}=0}^{PDG} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta_{23} & -\sin 2\theta_{23} \\ 0 & -\sin 2\theta_{23} & -\cos 2\theta_{23} \end{pmatrix} M_{ee}^{PDG}, \quad (18)$$

for the degenerate mass pattern with $\tilde{m}_1 \approx \tilde{m}_2 \approx -\tilde{m}_3$ (DMP) [21].¹

Applying these estimates to Eq.(12), we reach

1. for NMH, ignoring $M_{ee, e\mu, e\tau}^{PDG}$,

$$\delta_{CP} \approx \arg (M_{\mu\mu}^{PDG*} \delta M_{e\tau}^{PDG}), \quad (19)$$

¹ Since $M_{\mu\tau}^{PDG}$ does not vanish in the limit of $\tilde{m}_1 = \tilde{m}_2 = \tilde{m}_3$ because of the presence of $s_{13}e^{i\delta_{CP}}$, the simplest case of $\tilde{m}_1 \approx \tilde{m}_2 \approx \tilde{m}_3$ requiring fairly suppressed magnitude of $M_{\mu\tau}^{PDG}$ is not relevant. In other cases with $\tilde{m}_1 \approx -\tilde{m}_2$, relations among masses are complicated and seem to give no positive feedback to our discussions.

2. for IMH-1, ignoring $M_{e\mu, e\tau}^{PDG}$,

$$\delta_{CP} \approx \arg(M_{ee}^{PDG} \delta M_{e\tau}^{PDG*}) + \pi, \quad (20)$$

3. for IMH-2, ignoring $M_{\mu\mu, \mu\tau, \tau\tau}^{PDG}$,

$$\delta_{CP} \approx \arg(M_{ee}^{PDG} \delta M_{e\tau}^{PDG*} - t_{23} M_{e\mu}^{PDG} \delta M_{\tau\tau}^{PDG*}) + \pi, \quad (21)$$

4. for DMP, ignoring $M_{e\mu, e\tau}^{PDG}$,

$$\delta_{CP} \approx \arg \left[\left(\frac{1}{t_{23}} M_{\mu\tau}^{PDG*} + M_{\mu\mu}^{PDG*} \right) \delta M_{e\tau}^{PDG} + M_{ee}^{PDG} \delta M_{e\tau}^{PDG*} \right] (+\pi), \quad (22)$$

with an extra π for the inverted mass ordering.

It is thus concluded that the main source of δ_{CP} is $\delta M_{e\tau}^{PDG}$ except for IMH-2. This conclusion is in accord with the expectation from Eq.(9) that $\delta M_{\tau\tau}^{PDG}$ is suppressed unless $\tilde{m}_1 \approx -\tilde{m}_2$ as in IMH-2. Since $\arg(M_{e\mu}^{PDG}) = \arg(M_{e\tau}^{PDG})$ is valid for $\sin\theta_{13} = 0$, we expect that $\arg(M_{e\mu}^{PDG}) \approx \arg(M_{e\tau}^{PDG})$ is preserved for $\sin\theta_{13} \neq 0$ especially in NMH because the single term proportional to \tilde{m}_3 will dominate in $M_{e\mu, e\tau}^{PDG}$. Using the approximation of $\arg(\delta M_{e\tau}^{PDG}) = \arg(M_{e\tau}^{PDG} + t_{23} M_{e\mu}^{PDG}) \approx \arg(M_{e\mu, e\tau}^{PDG})$, we can find more simplified relation from Eq.(19) in NMH:

$$\delta_{CP} \approx \arg(M_{e\mu}^{PDG}) - \arg(M_{\mu\mu}^{PDG}), \quad (23)$$

with $\arg(M_{e\mu}^{PDG}) \approx \arg(M_{e\tau}^{PDG})$. The similar relation is also found for IMH-1 and dictates from Eq.(20) that

$$\delta_{CP} \approx \arg(M_{ee}^{PDG}) - \arg(M_{e\tau}^{PDG}) + \pi, \quad (24)$$

where $\arg(M_{e\mu}^{PDG}) \approx \arg(M_{e\tau}^{PDG})$ does not serve as a good approximation. In fact, Eq.(24) using another choice of $\arg(M_{e\mu}^{PDG})$ instead of $\arg(M_{e\tau}^{PDG})$ is not numerically supported (See FIG.2-(a)).

To further enhance predictability based on our approach to CP-violations, we have to minimize the number of phases present in flavor neutrino masses, which can be as small as three. Therefore, a plausible program to discuss linkage between CP-violating phases and flavor neutrino masses is

1. to construct a reference mass matrix to be denoted by M_ν with unique choice of phases of neutrino masses,
2. to construct a general mass matrix to be denoted by M that includes the ambiguity of charged-lepton phases to cover all phase structure, which is linked to M_ν ,
3. to construct M^{PDG} converted from M , whose eigenvectors yield U_{PMNS}^{PDG} .

Since flavor neutrino masses in M^{PDG} are expressed by measured quantities, useful information on phases of M_ν can be extracted from M^{PDG} .

We start with the following neutrino mass matrix M_ν , which has three complex flavor neutrino masses $M_{e\mu}$, $M_{e\tau}$ and $M_{\tau\tau}$. This choice of phases is suggested by Eq.(7) and yields

$$M_\nu = \begin{pmatrix} |M_{ee}| & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & |M_{\mu\mu}| & |M_{\mu\tau}| \\ M_{e\tau} & |M_{\mu\tau}| & M_{\tau\tau} \end{pmatrix}. \quad (25)$$

The mass matrix M physically equivalent to M_ν can be obtained by including the freedom of three charged-lepton phases denoted by $\theta_{e,\mu,\tau}$ and is expressed to be:

$$M = \begin{pmatrix} e^{-2i\theta_e} |M_{ee}| & e^{-i(\theta_e+\theta_\mu)} M_{e\mu} & -e^{-i(\theta_e+\theta_\tau)} M_{e\tau} \\ e^{-i(\theta_e+\theta_\mu)} M_{e\mu} & e^{-2i\theta_\mu} |M_{\mu\mu}| & e^{-i(\theta_\mu+\theta_\tau)} |M_{\mu\tau}| \\ -e^{-i(\theta_e+\theta_\tau)} M_{e\tau} & e^{-i(\theta_\mu+\theta_\tau)} |M_{\mu\tau}| & e^{-2i\theta_\tau} M_{\tau\tau} \end{pmatrix}. \quad (26)$$

One has to diagonalize Eq.(26) to give $m_{1,2,3}$. Since Eq.(26) contains six phases associated with six complex masses, the relevant U_{PMNS} , U'_{PMNS} , should contain six phases, among which three phases are redundant [22, 23]. We use three phases denoted by δ associated with the 1-3 mixing, γ associated with the 2-3 mixing and ρ associated with

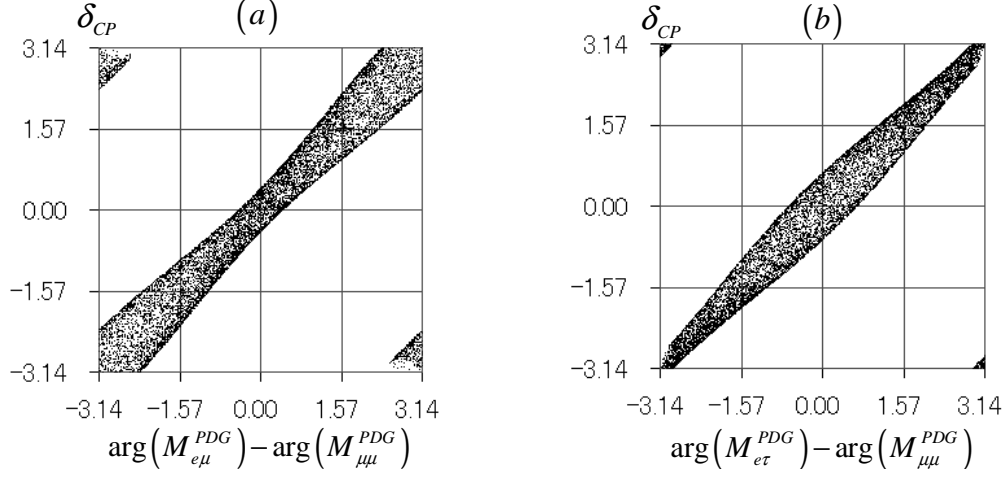


FIG. 1: The predictions of δ_{CP} for the normal mass hierarchy (NMH): (a) $\delta_{CP} \approx \arg(M_{e\mu}^{PDG}) - \arg(M_{\mu\mu}^{PDG})$ or (b) $\delta_{CP} \approx \arg(M_{e\tau}^{PDG}) - \arg(M_{\mu\mu}^{PDG})$.

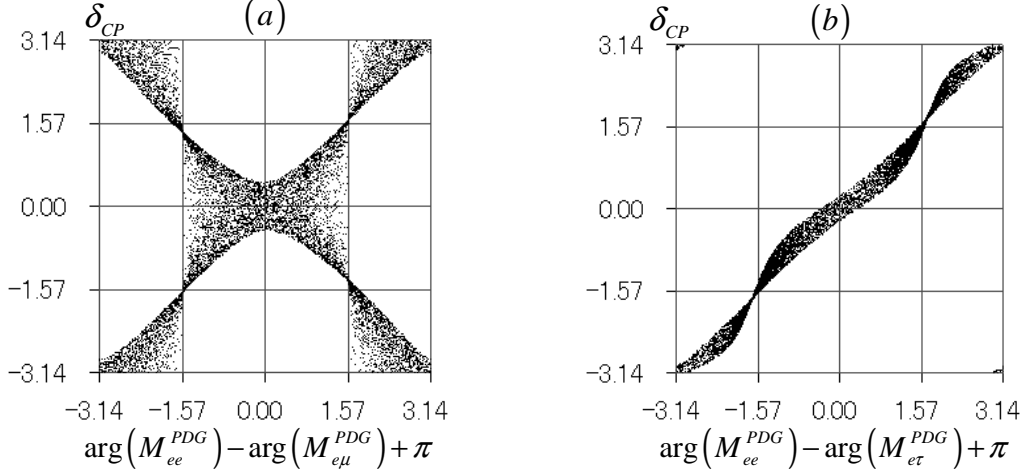


FIG. 2: The predictions of δ_{CP} for the inverted mass hierarchy with $\tilde{m}_1 \approx \tilde{m}_2$ (IMH-1): (a) $\delta_{CP} \approx \arg(M_{ee}^{PDG}) - \arg(M_{e\mu}^{PDG}) + \pi$ or (b) $\delta_{CP} \approx \arg(M_{ee}^{PDG}) - \arg(M_{e\tau}^{PDG}) + \pi$.

the 1-2 mixing and another three phases denoted by $\alpha_{1,2,3}$ as Majorana phases to define U'_{PMNS} [22]. After three redundant phases ρ , γ and φ_1 are removed from U'_{PMNS} , Eq.(26) is modified into:

$$M^{PDG} = \begin{pmatrix} e^{2i\rho_e} |M_{ee}| & e^{i(\rho_e + \gamma_\mu)} M_{e\mu} & e^{i(\rho_e - \gamma_\tau)} M_{e\tau} \\ e^{i(\rho_e + \gamma_\mu)} M_{e\mu} & e^{2i\gamma_\mu} |M_{\mu\mu}| & e^{i(\gamma_\mu - \gamma_\tau)} |M_{\mu\tau}| \\ e^{i(\rho_e - \gamma_\tau)} M_{e\tau} & e^{i(\gamma_\mu - \gamma_\tau)} |M_{\mu\tau}| & e^{-2i\gamma_\tau} M_{\tau\tau} \end{pmatrix}, \quad (27)$$

where $\rho_e = \rho - \theta_e$, $\gamma_\mu = \gamma - \theta_\mu$ and $\gamma_\tau = \gamma + \theta_\tau$, which can be diagonalized by U_{PMNS} of Eq.(1) with $\delta_{CP} = \delta + \rho$, $\phi_2 = \varphi_2 - \varphi_1$ and $\phi_3 = \varphi_3 - \varphi_1$ for $\varphi_1 = \alpha_1 - \rho$ and $\varphi_{2,3} = \alpha_{2,3}$. As a result, phases of $M_{e\mu}$, $M_{e\tau}$ and $M_{\tau\tau}$ are expressed in terms of $\arg(M_{ij}^{PDG})$ ($i, j = e, \mu, \tau$) as follows:

$$\begin{aligned} \arg(M_{e\mu}) &= \arg(M_{e\mu}^{PDG}) - \frac{\arg(M_{ee}^{PDG}) + \arg(M_{\mu\mu}^{PDG})}{2}, \\ \arg(M_{e\tau}) &= \arg(M_{e\tau}^{PDG}) - \frac{\arg(M_{ee}^{PDG}) - \arg(M_{\mu\mu}^{PDG})}{2}, \\ \arg(M_{\tau\tau}) &= \arg(M_{\tau\tau}^{PDG}) + \arg(M_{\mu\mu}^{PDG}) - 2\arg(M_{\mu\tau}^{PDG}). \end{aligned} \quad (28)$$

Our results of numerical calculations are listed in FIG.1-FIG.6. Shown in FIG.1 and FIG.2 are predictions on δ_{CP}

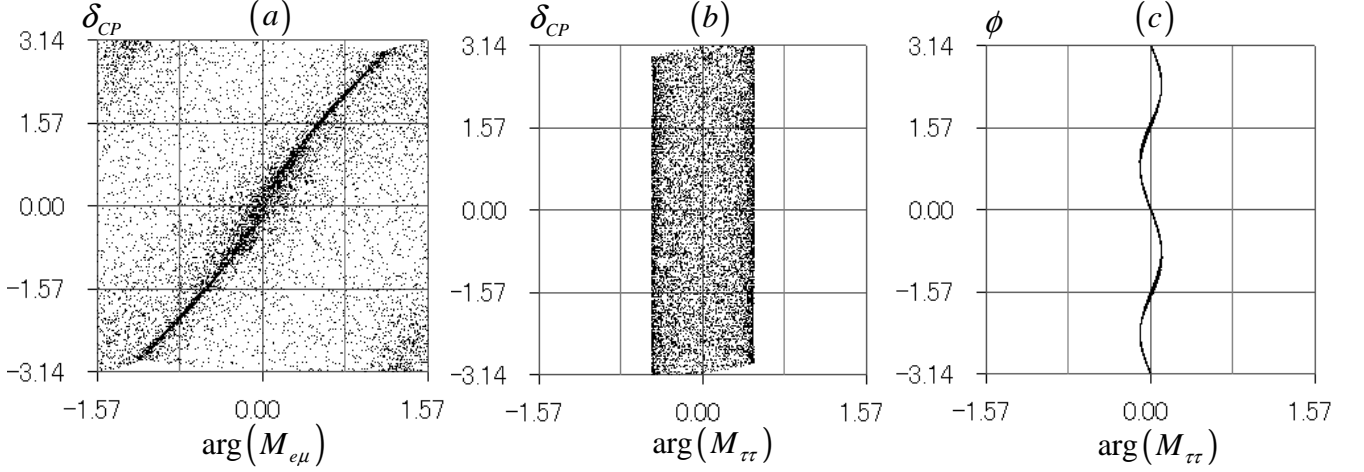


FIG. 3: The predictions of (a) δ_{CP} as a function of $\arg(M_{e\mu})$, (b) δ_{CP} as a function of $\arg(M_{\tau\tau})$ and (c) ϕ ($=\varphi_3 - \varphi_2$) as a function of $\arg(M_{\tau\tau})$ for the normal mass hierarchy (NMH).

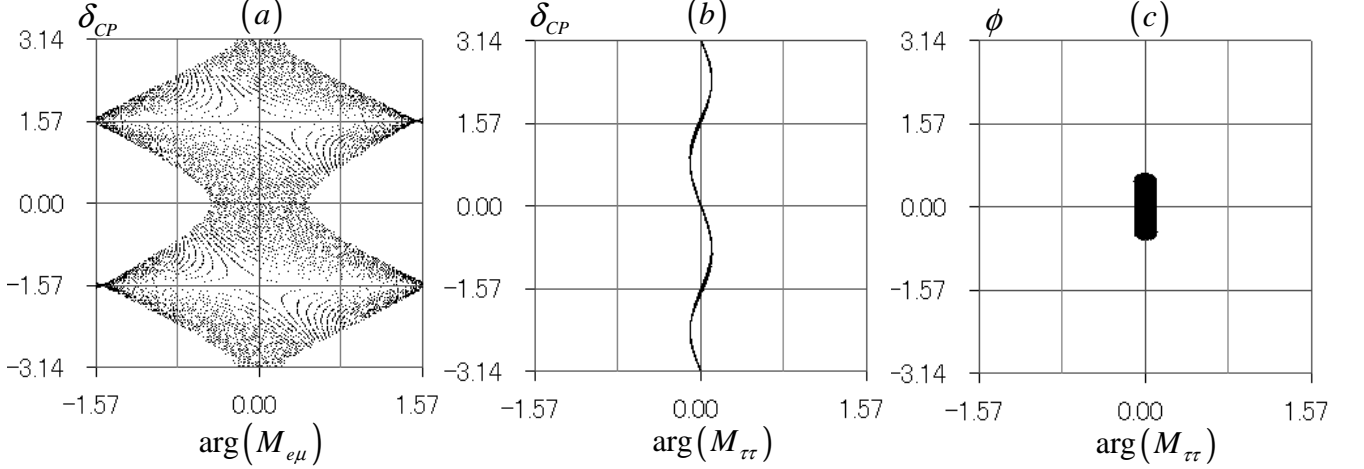


FIG. 4: The same as in FIG.3 but for the inverted mass hierarchy with $\tilde{m}_1 \approx \tilde{m}_2$ (IMH-1) and $\phi = \varphi_2 - \varphi_1$.

from the simplified relations Eqs.(23) and (24). In the remaining figures, FIG.3-FIG.6, each of which corresponds to each mass pattern, predictions on δ_{CP} are depicted as functions of either $\arg(M_{e\mu})$ or $\arg(M_{\tau\tau})$, which exhibit a certain correlation to δ_{CP} . On the other hand, any predominant correlation between δ_{CP} and $\arg(M_{e\tau})$ in each case cannot be found. Other correlations to CP-violating Majorana phases are also shown in the figures. For the sake of simplicity, calculations have been done for $m_1 = 0$ eV for NMH and $m_3 = 0$ eV for IMH-1 and IMH-2. For DMP, $m_1=0.1$ eV ($m_3=0.1$ eV) for the normal (inverted) mass ordering is adopted. The parameters used are

$$\Delta m_{21}^2 [10^{-5} \text{ eV}^2] = 7.62, \quad \Delta m_{31}^2 [10^{-3} \text{ eV}^2] = 2.53, \quad (29)$$

$$\sin^2 \theta_{12} = 0.32, \quad \sin^2 \theta_{23} = 0.45, \quad \sin^2 \theta_{13} = 0.025. \quad (30)$$

The suggested values of δ_{CP}/π at the 1σ range are $0.83^{+0.54}_{-0.64}$ for normal mass ordering [10] or 0.45-1.18 for normal mass ordering and 0.47-1.22 for inverted mass ordering [11].

As can be seen from FIG.1 and FIG.2, it is observed that the figures indicate the approximate proportionality of δ_{CP} to predicted values of Eq.(23) for δ_{CP} using both $\arg(M_{e\mu}^{PDG})$ and $\arg(M_{e\tau}^{PDG})$ and of Eq.(24) using $\arg(M_{e\tau}^{PDG})$, which supports the validity of our predictions. However, FIG.2-(a) for IMH shows that δ_{CP} using $\arg(M_{e\mu}^{PDG})$ is not a suitable approximation and implies that the assumption of $\arg(M_{e\mu}^{PDG}) \approx \arg(M_{e\tau}^{PDG})$ is not numerically supported. From Eqs.(23) and (24), we obtain $\arg(M_{e\mu, e\tau})$ related to δ_{CP} as

$$\arg(M_{e\mu}) \approx \delta_{CP} - [\arg(M_{ee}^{PDG}) - \arg(M_{\mu\mu}^{PDG})] / 2, \quad (31)$$

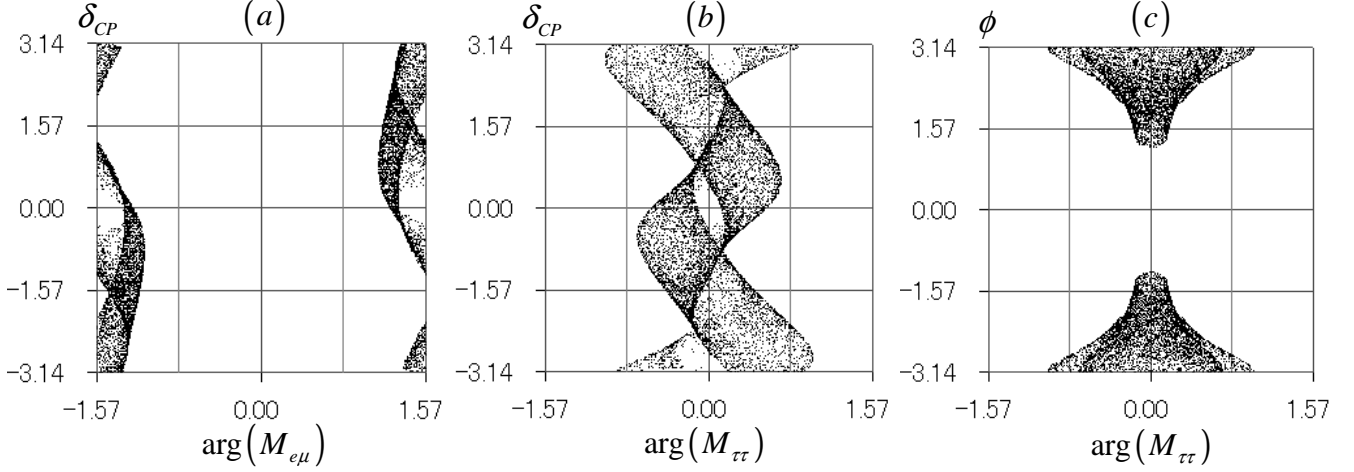


FIG. 5: The same as in FIG.3 but for the inverted mass hierarchy with $\tilde{m}_1 \approx -\tilde{m}_2$ (IMH-2).

for NMH, and

$$\arg(M_{e\tau}) \approx -\delta_{CP} + [\arg(M_{ee}^{PDG}) + \arg(M_{\mu\mu}^{PDG})]/2 + \pi, \quad (32)$$

for IMH-1. We have also checked that the approximated expressions of δ_{CP} , Eqs.(19)-(22), numerically well reproduce actual values of δ_{CP} .

For the results of $\arg(M_{e\mu, \tau\tau})$ with respect to δ_{CP} and CP-violating Majorana phases, their suggested features are summarized as follows:

- For NMH with $m_1 = 0$, where $\phi = \varphi_3 - \varphi_2$, FIG.3 indicates that
 - δ_{CP} following the thick line approximated to be $\delta_{CP} \approx 2 \arg(M_{e\mu})$ is realized by requiring $\arg(M_{ee}^{PDG}) - \arg(M_{\mu\mu}^{PDG}) \approx \delta_{CP}$ because of Eq.(31),
 - $|\arg(M_{\tau\tau})| \lesssim 0.5$,
 - ϕ has a simple dependence on $\arg(M_{\tau\tau})$: $\phi = 0, \pi$ if $\arg(M_{\tau\tau}) = 0$ and $\phi = \pm\pi/2$ if $|\arg(M_{\tau\tau})|$ reaches its maximal value of around 0.5.
- For IMH-1, where $\phi = \varphi_2 - \varphi_1$, FIG.4 indicates that
 - $\delta_{CP} \rightarrow \pm\pi/2$ as $\arg(M_{e\mu}) \rightarrow \pm\pi/2$,
 - $|\arg(M_{\tau\tau})| \lesssim 0.1$,
 - $\phi \approx 0$ is set by the condition of $\tilde{m}_1 \approx \tilde{m}_2$.
- For IMH-2, where $\phi = \varphi_2 - \varphi_1$, FIG.5 indicates that
 - $\pi/3 \lesssim |\arg(M_{e\mu})| \lesssim \pi/2$,
 - $|\arg(M_{\tau\tau})| \lesssim 0.6$ for $|\delta_{CP}| \lesssim \pi/2$,
 - $|\arg(M_{\tau\tau})| \lesssim 0.2$ if ϕ approaches toward $\pm\pi/2$.
 - $\phi \approx \pm\pi$ is set by the condition of $\tilde{m}_1 \approx -\tilde{m}_2$.
- For DMP, FIG.6 indicates that
 - $\pi/4 \lesssim |\delta_{CP}| \lesssim 3\pi/4$ as $\arg(M_{e\mu}) \rightarrow \pm\pi/2$ for the normal mass ordering,
 - $\delta_{CP} \rightarrow \pm\pi/2$ as $\arg(M_{e\mu}) \rightarrow \pm\pi/2$ for the inverted mass ordering,
 - $\phi_2 \approx 0$ and $\phi_3 \approx \pm\pi$ for both mass orderings, which are linked to the fact that the sign of \tilde{m}_3 is different from that of $\tilde{m}_{1,2}$.

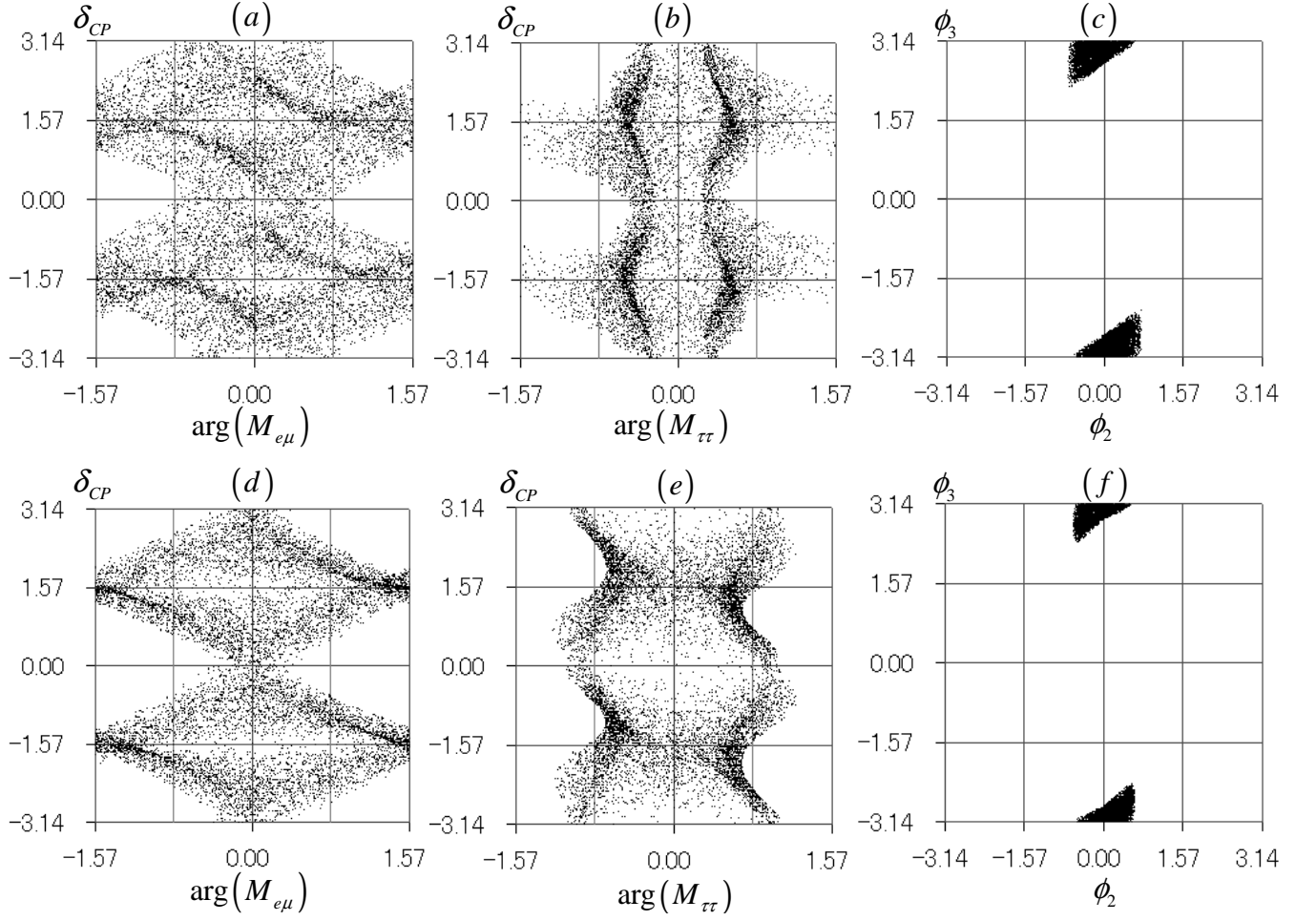


FIG. 6: The same as in FIG.3 for (a) and (b) but (c) ϕ_3 as a function of ϕ_2 for the degenerate mass pattern with $\tilde{m}_1 \approx \tilde{m}_2 \approx -\tilde{m}_3$ (DMP) satisfying the normal mass ordering. Lower figures (d)-(f) show results for the inverted mass ordering.

The phases of $M_{e\mu, \tau\tau}$ are taken to run from $-\pi/2$ to $\pi/2$. It should be noted that $\arg(M_{e\mu})$ - δ_{CP} for IMH-1 (FIG.4-(a)) and for DMP with the inverted mass ordering (FIG.6-(a)) have the quite similar shape to each other, showing that the maximal Dirac CP-violation signalled by $\delta_{CP} = \pm\pi/2$ is realized by $\arg(M_{e\mu}) \approx \pm\pi/2$ and, at the same time, $\arg(M_{\tau\tau}) \approx 0$ is necessary for IMH-1.

Also estimated is $|M_{ee}|$ as the effective neutrino mass $m_{\beta\beta}$ in $(\beta\beta)_{0\nu}$ -decay [24]:

- $|M_{ee}|$ [eV] ≈ 0.01 for NMH
- $|M_{ee}|$ [eV] ≈ 0.05 for IMH-1
- $0.01 \lesssim |M_{ee}|$ [eV] $\lesssim 0.05$ for IMH-2
- $0.095 \lesssim |M_{ee}|$ [eV] $\lesssim 0.1$ for DMP.

The results are consistent with naive estimation from Eqs.(15)-(18). Namely, the magnitude of $m_{\beta\beta}$ is suppressed for NMH.

To summarize, we have derived a general formula to calculate δ_{CP} expressed in terms of the corrections $\delta M_{e\tau}^{PDG}$ and $\delta M_{\tau\tau}^{PDG}$ to neutrino mixings with $\theta_{13} = 0$. The formula is given by Eq.(12):

$$\delta_{CP} \approx \arg \left[\left(\frac{1}{t_{23}} M_{\mu\tau}^{PDG*} + M_{\mu\mu}^{PDG*} \right) \delta M_{e\tau}^{PDG} + M_{ee}^{PDG} \delta M_{e\tau}^{PDG*} - t_{23} M_{e\mu}^{PDG} \delta M_{\tau\tau}^{PDG*} \right], \quad (33)$$

where an extra π should be added if $m_3^2 - (c_{12}^2 m_1^2 + s_{12}^2 m_2^2) < 0$. These $\delta M_{e\tau}^{PDG}$ and $\delta M_{\tau\tau}^{PDG}$ are described in terms of M^{PDG} as $\sin\theta_{13}\delta M_{e\tau}^{PDG} = M_{e\tau}^{PDG} + t_{23}M_{e\mu}^{PDG}$ and $\sin\theta_{13}\delta M_{\tau\tau}^{PDG} = M_{\tau\tau}^{PDG} - [M_{\mu\mu}^{PDG} + (1 - t_{23}^2)M_{\mu\tau}^{PDG}/t_{23}]$. Their mass dependence is then determined to be:

$$\delta M_{e\tau}^{PDG} = \frac{c_{13}}{c_{23}} [e^{i\delta_{CP}} \tilde{m}_3 - e^{-i\delta_{CP}} (c_{12}^2 \tilde{m}_1 + s_{12}^2 \tilde{m}_2)], \quad \delta M_{\tau\tau}^{PDG} = \frac{c_{12}s_{12}}{s_{23}c_{23}} e^{-i\delta_{CP}} (\tilde{m}_2 - \tilde{m}_1). \quad (34)$$

Other useful findings are

- The main source of δ_{CP} is $\delta M_{e\tau}^{PDG}$ except for IMH-2 because $\delta M_{\tau\tau}^{PDG}$ is not suppressed if $\tilde{m}_1 \approx -\tilde{m}_2$, and
- δ_{CP} is well predicted to be $\arg(M_{e\mu}^{PDG}) - \arg(M_{\mu\mu}^{PDG})$ with $\arg(M_{e\tau}^{PDG}) \approx \arg(M_{e\mu}^{PDG})$ for NMH and $\arg(M_{ee}^{PDG}) - \arg(M_{e\tau}^{PDG}) + \pi$ for IMH-1.

For the specific neutrino masses, whose phases are adjusted to arise from $M_{e\mu, e\tau, \tau\tau}$, the effects of CP-violation caused by each flavor neutrino mass are expressed in terms of M^{PDG} according to Eq.(28). For the numerical calculations, we adopted $m_1 = 0$ eV ($m_3 = 0$ eV) for NMH (IMH) and $m_1 = 0.1$ eV ($m_3 = 0.1$ eV) for DMP with the normal (inverted) mass ordering. It is, then, numerically indicated that δ_{CP} tends to satisfy $\delta_{CP} \approx 2 \arg(M_{e\mu})$ requiring the relation of $\arg(M_{ee}^{PDG}) - \arg(M_{\mu\mu}^{PDG}) \approx \delta_{CP}$ in NMH. In the inverted mass hierarchies, we have observed that $|\arg(M_{\tau\tau})| \lesssim 0.1$ for IMH-1 and $\pi/3 \lesssim |\arg(M_{e\mu})| \lesssim \pi/2$ for IMH-2. CP-violating Majorana phase ϕ_2 (ϕ_3) for DMP is limited to locate around 0 ($\pm\pi$) owing the mass relation of $\tilde{m}_1 \approx \tilde{m}_2 \approx -\tilde{m}_3$. Effects of Majorana CP-violation are expected to be suppressed for DMP. On the other hand, for NMH, Majorana CP-violation tends to be maximal as $|\arg(M_{\tau\tau})|$ reaches its maximal value of ≈ 0.5 . If Majorana CP-violation tends to be maximal, we have also found that $|\arg(M_{\tau\tau})| \lesssim 0.2$ for IMH-2. Dirac CP-violation gets maximal as $\arg(M_{e\mu}) \rightarrow \pm\pi/2$ for IMH-1 and DMP with the inverted mass ordering and $\arg(M_{\tau\tau}) \approx 0$ is also satisfied for IMH-1.

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